Twistor superstring in two-time physics

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By utilizing the gauge symmetries of two-time physics (2T physics), a superstring with linearly realized global SU(2, 2|4) supersymmetry in $4+2$ dimensions (plus internal degrees of freedom) is constructed. It is shown that the dynamics of the Witten-Berkovits twistor superstring in $3+1$ dimensions emerges as one of the many one-time (1T) holographic pictures of the $4+2$ dimensional string obtained via gauge fixing of the 2T gauge symmetries. In 2T physics the twistor language can be transformed to usual spacetime language and vice versa, off shell, as different gauge fixings of the same 2T string theory. Further holographic string pictures in $3+1$ dimensions that are dual theories also can be derived. The 2T superstring is further generalized in the SU(4) = SO(6) sector of SU(2, 2|4) by the addition of six bosonic dimensions, for a total of $10+2$ dimensions. Excitations of the extra bosons produce a SU(2, 2|4) current algebra spectrum that matches the classification of the high-spin currents of $N=4$, $d=4$ super Yang-Mills theory which are conserved in the weak coupling limit. This spectrum is interpreted as the extension of the SU(2, 2|4) classification of the Kaluza-Klein towers of typeII-B supergravity compactified on $AdS_5 \times S^5$, into the full string theory, and is speculated to have a covariant $10+2$ origin in F-theory or S-theory. Further generalizations of the superstring theory to $3+2$, $5+2$, and $6+2$ dimensions based on the supergroups OSp(8|4), F(4), OSp(8^*|4), respectively, and other cases, are discussed also. The OSp(8^*|4) case in $6+2$ dimensions can be gauge fixed to $5+1$ dimensions to provide a formulation of the special superconformal theory in six dimensions either in terms of ordinary spacetime or in terms of twistors.

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I. 2T FORMULATION

The Witten [1,2] and Berkovits [3,4] twistor superstring, or the corresponding $N=4$, $d=4$ super Yang-Mills, and superconformal gravity theories [5], are invariant under SO(4,2) conformal symmetry and its supersymmetric generalization SU(2, 2|4). The conformal symmetry SO(d, 2) is a hint for two-time physics (2T physics) [6] in *flat* $d+2$ dimensional spacetime. In this paper it will be shown that the twistor superstring is a gauge fixed version of a 2T superstring in $4+2$ dimensions.

The first aim of the present paper is to apply to the Witten-Berkovits twistor superstring the consequence of 2T physics, which is the discovery of dually related string models and the establishment of duality type relations among them, while displaying an underlying spacetime with one extra space and one extra time dimension that unify the various dually related holographic pictures as a single parent 2T theory.

Once this fact is established, the second aim of this paper is to propose generalizations of the theory to $d+2$ dimensions. This works only in special dimensions. The cases of $3+2$, $4+2$, $5+2$, $6+2$ are direct generalizations of the $4+2$ case and like the $4+2$ case also describe super conformal theories in dimensions $d=3, 4, 5, 6$ as discussed later in the paper. Another type of generalization of $4+2$ to $10+2$ is obtained by the addition of six more dimensions to obtain a new 2T string theory in $10+2$ dimensions. In a particular gauge of the 2T theory the $10+2$ system reduces to a $9+1$ dimensional theory and describes a string in the $AdS_5 \times S^5$ background. The twistor version of this theory is also obtained. It is already known that the particle limit of the $10+2$ string theory gives the complete set of the Kaluza-Klein towers of type-IIB supergravity compactified on $AdS_5 \times S^5$ as shown in [7,8]. This is now generalized to the string version.

Although not yet fully analyzed, tentatively it appears that the full string spectrum of the $10+2$ theory has the SU(2, 2|4) quantum numbers that match the AdS-CFT correspondence to the conserved currents of $N=4$, $d=4$ super Yang-Mills theory in the weak coupling limit. The classification of the $10+2$ states under the little group SO(10) was suggested a long time ago for the T-dual M-theory version in $10+1$ dimensions [9] and is argued below that the SO(10) classification also applies to a $10+2$ theory via T-duality. The classification of the high-spin super Yang-Mills (SYM) currents at weak coupling was suggested recently in [10] by using group theoretical steps that are closely parallel to those previously used in [9].[1] Most significantly, the SYM spectrum of currents in [10] can be understood as the decomposition of $SO(10) \to SO(4) \times SO(6)$ applied on the higher dimensional spectrum in [9] (including Kaluza-Klein excitations) corresponding to the compactification $(10+2) \to (4+2)+(6+0)$ dimensions. The 2T superstring in $10+2$

[1]Note also that Polya theory as used in [10] is parallel to the concept of "color" as used in the oscillation representation of groups [8,11].

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dimensions in the current paper seems to provide the basis to explain this spectrum as belonging to a compactified version of F-theory [12] or S-theory [13,14], thus giving the source of this spectrum in string theory. Furthermore, the 2T superstring taken in a variety of 1T gauges yields a collection of 1T dynamical models as dual holographic pictures in $9+1$ dimensions, all of which have spectra related by dualities (changes of bases) within the same group theoretical representations.

The concepts discussed in this paper are based on technology developed previously in supersymmetric 2T physics for particles [15], the twistor gauge for 2T superparticles [16], the $10+2$ dimensional $SU(2,2|4)$ superparticle [7], the oscillator representations of supergroups [11] as refined recently [8], and strings in the 2T framework [17]. The salient aspects of the previous work will be reviewed below.

A general remark about 2T physics is that it contains a gauge symmetry $Sp(2,R)$ that, in flat spacetime,[2] acts on phase space (X^M, P^M) as a doublet for each M. The first class constraints $X^2 = P^2 = X \cdot P = 0$ due to this gauge symmetry have nontrivial solutions, and the theory is unitary and causal, only if target spacetime has two timelike dimensions, no more and no less. Then one finds that 2T physics is consistent with one-time physics (1T physics) in the $Sp(2,R)$ gauge invariant sector, or after the removal of gauge degrees of freedom. Thus, the 2 times in target space arise as a consequence of the $Sp(2,R)$ gauge symmetry, they are not an input. Then $SO(d,2)$ is the global symmetry acting on the spacetime M index in flat space. The $SO(d,2)$ symmetry commutes with the local $Sp(2,R)$ hence it is gauge invariant and classifies the physical spectrum.

Another general remark about 2T physics is that many one-time physics systems emerge via gauge fixing the 2T system. The 2T action naturally unifies 1T systems that are dually related among themselves, because they are all related to the same parent 2T system via the $Sp(2,R)$ gauge transformations. Thus 1T systems, which may appear unrelated in the absence of the understanding reached via 2T physics, get unified through the higher dimensional 2T theory. The gauge fixing from 2T to 1T is done by using two out of the three local parameters of the $Sp(2,R)$ symmetry, plus two out of the three corresponding constraints, to reduce the phase space degrees of freedom by one timelike and one spacelike degrees of freedom. The remaining phase space has $(d-1)$ spacelike and 1 timelike dimensions and provides a holographic description of the higher dimensional 2T system in the reduced phase space. There are many possible holographic pictures of the higher system corresponding on how the remaining timelike coordinate is embedded in the $d+2$ higher spacetime.

[2]The general case in the presence of backgrounds is developed in [18]

The choice of the remaining 1-timelike coordinate rearranges the dynamics of the 2T system to evolve according to that choice of time. This leads to different Hamiltonians to describe each of the 1T holographic pictures. Therefore, each distinct gauge choice of time makes the same 2T system appear as distinct dynamics from the point of view of 1T physics. The holographic pictures obtained from the same 2T action are related to one another by duality type relations, where the duality transformation is a $Sp(2,R)$ gauge transformation (which is nonlinearly realized on the remaining phase space once a gauge is chosen). Many striking examples of this phenomenon have been displayed in simple classical and quantum mechanics. Some of the simplest examples include free relativistic particle, its twistor description, nonrelativistic particle, hydrogen atom, harmonic oscillator, particle on $AdS_{d-k} \times S^k$ background, etc., all being holographic pictures of the same 2T *free particle*. One can directly verify that these systems are indeed related as predicted by 2T physics [6,19]. These simple examples, and spinning and supersymmetric generalizations, with or without background fields [6,18], already establish the existence of 2T physics as a solid framework that describes reality.

The global symmetry $SO(d,2)$ in flat spacetime is linearly realized in the 2T phase space (X^M, P^M). In the present paper M is labeled as $M = [0', 0, 1', 1, 2, \cdots, (d-1)]$, or $M = (+', -', \mu)$ where $+', -'$ are lightcone type combinations constructed from the directions $0', 1'$, while $\mu = [0, 1, \cdots, (d-1)]$ is a d-dimensional Lorentz index.

II. 2T SUPERPARTICLE IN 4 + 2

The 2T formulation of the superparticle in $d = 3, 4, 5, 6$ with N supersymmetries is introduced in [15] and further developed in [16], while the extension with more bosonic dimensions is discussed in [7]. These will be directly relevant for the 2T reformulation of the twistor superstring. For this purpose, first we recall the superparticle in $d = 4$ dimensions with $N = 4$ supersymmetries. It requires $4+2$ coordinates $X^M(\tau)$ and momenta $P^M(\tau)$, and a supergroup element $g(\tau) \in SU(2,2|4)$ that contains fermions $\Theta_s^a(\tau)$ in the off-diagonal blocks, where (s, a) denote the complex bifundamental representation (4,4) of $SU(2,2) \times SU(4)$. This spinor has double the size of the smallest spinor $\theta_\alpha^a(\tau)$ in four dimensions, which is of course necessary if the $SO(4,2) = SU(2,2)$ is to be realized linearly in $4+2$ dimensions. Thus, compared to the 1T formulation there are extra degrees of freedom in X, P, Θ and in the $SU(2,2) \times SU(4)$ bosonic blocks in $g(\tau)$. If the covariant 2T formulation of the superparticle is to be equivalent to the 1T formulation there has to be various gauge symmetries and extended kappa supersymmetries to cut down the degrees of freedom to the correct set for the superparticle in $d = 4$ and $N = 4$. As shown in [15,16] this is beautifully achieved with the following action

$$\mathcal{L}_{2T} = \frac{1}{2}\varepsilon^{ij}\partial_\tau X_i \cdot X_j - \frac{1}{2}A^{ij}X_i \cdot X_j + \frac{1}{2}\mathrm{Str}(i\partial_\tau g g^{-1} L),$$
$$L \equiv \begin{pmatrix} \frac{i}{2}\Gamma_{MN}L^{MN} & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.1}$$

where $X_i^M = (X^M, P^M)$, with $i = 1, 2$ is the Sp(2, R) doublet; ε^{ij} is the antisymmetric invariant metric of Sp(2, R); the symmetric $A^{ij} = A^{ji}$ is the Sp(2, R) gauge potential; Γ_M and $\Gamma_{MN} = \frac{1}{2}[\Gamma_M, \Gamma_N]$ are SO(4,2) gamma matrices in the spinor representation embedded in the first 4×4 SU(2, 2) block of the SU(2, 2|4) matrix. The Cartan connection $i\partial_\tau g g^{-1}$ projected in the direction of the subgroup SO(4, 2)SU(2, 2|4) is coupled to the SO(4,2) orbital angular momentum $L^{MN} = \varepsilon^{ij}X_i^M X_j^N = X^M P^N - X^N P^M$ which is Sp(2, R) gauge invariant. Note that this SO(4,2) connection is not a pure gauge since the 16 complex fermionic coset parameters Θ_s^a in SU(2, 2|4)/[SU(2, 2) $\times$ SU(4)] contribute to it.

This action has a *global* $\mathrm{SU}(2,2|4)_R$ supersymmetry that acts on the right side of g, namely, $g(\tau) \to g(\tau)g_R$. It also has *local* bosonic SU(2, 2) $\times$ SU(4) and local fermionic kappa supersymmetries embedded in $\mathrm{SU}(2,2|4)_L$ that act simultaneously on the left side[3] of g, namely, $g(\tau) \to g_L(\tau)g(\tau)$, as well as on the phase space (X^M, P^M) and the gauge fields A^{ij} [see Eqs. (3.4) to (3.15)].

We discuss below three approaches to the quantization of this 2T superparticle system: covariant quantization in $4 + 2$ dimensions, 1T superparticle gauge in $3 + 1$ dimensions and its quantization, and the supertwistor gauge and its quantization. In every approach the physical quantum states correspond to the physical degrees of freedom of $N = 4$, $d = 4$ SYM theory.

The conserved Noether charges for the global $\mathrm{SU}(2,2|4)_R$ symmetry are the entries of the 8×8 supermatrix[4]

$$J_A{}^B = \left(\frac{1}{4}g^{-1}Lg\right)_A{}^B, \qquad \partial_\tau J_A{}^B(\tau) = 0. \tag{2.2}$$

These charges are invariant under all of the gauge symmetries, namely, Sp(2, R) as well as the ones embedded in $\mathrm{SU}(2,2|4)_L$ (more details on this when we discuss the string below). Therefore they define the physical states of the system as representations of $\mathrm{SU}(2,2|4)_R$. Let us analyze some properties of J. The square of this matrix is $J^2 = \frac{1}{16}g^{-1}L^2 g$. At the classical level L^2 vanishes after using the Sp(2, R) constraints $X_i \cdot X_j = 0$ that follow from the action (namely when one considers the Sp(2, R) gauge invariant sector of phase space X^M, P^M). At the quantum level, we use the commutation rules of L^{MN} to compute $L^2 = (D-2)L + \mathrm{diag}(\frac{1}{2}L^{MN}L_{MN}\mathbf{1}_{2,2}, \mathbf{0}_4)$, including the linear term. Thus, at the quantum level, instead of vanishing J^2 we obtain for $D = 6$ the following projector equation

$$(J^2)_A{}^B = J_A{}^B, \tag{2.3}$$

on the Sp(2, R) gauge invariant quantum states.[5] This gives the $\mathrm{SU}(2,2|4)_R$ covariant quantization of Eq. (2.1), by identifying the physical states as those that satisfy the condition $J^2 = J$ for the $\mathrm{SU}(2,2|4)_R$ charges. The supersingleton of $\mathrm{SU}(2,2|4)_R$ satisfies this condition (this was first realized in the context of 2T physics in [7,8]). Furthermore, it is well known that the spectrum of the SU(2, 2|4) supersingleton corresponds precisely to the fields of the linearized $N = 4$, $d = 4$ super Yang-Mills theory and all their derivatives [11] [8]. Hence the physical states of Eq. (2.1) are described by the SYM fields.

It is possible also to obtain the supersingleton spectrum by choosing some gauge which gives a holographic picture in $3 + 1$ dimensions. The action Eq. (2.1) has just the required gauge symmetries to gauge fix the 2T superparticle into various holographic pictures that describe 1T physics. One holographic picture is the standard superparticle in four dimensions

$$\mathcal{L}_{\text{particle}} = \dot{x} \cdot p - \frac{1}{2}A^{22}p^2 + \bar{\theta}_a \gamma^\mu \partial_\tau \theta^a p_\mu$$
$$\leftrightarrow \frac{1}{2A^{22}}(\dot{x}^\mu + \bar{\theta}_a \gamma^\mu \partial_\tau \theta^a)^2, \tag{2.4}$$

with $\mu = 0, 1, 2, 3$, and θ_α^a four complex SL(2, C) doublets. This holographic picture is generated by (1) using a local symmetry SU(2, 2) $\times$ SU(4)$\subset$ $\mathrm{SU}(2,2|4)_L$ to remove all the bosonic degrees of freedom in $g(\tau)$; (2) partially fixing the fermionic kappa symmetry in $\mathrm{SU}(2,2|4)_L$ to cut down the original 16 complex fermions Θ_s^a in g by a factor of 2 to eight complex fermions θ_α^a, i.e., $\alpha = 1, 2$; $a = 1, 2, 3, 4$ (with leftover kappa symmetry), so that $g(\tau)$ takes the form

$$g = \exp\begin{pmatrix} 0_2 & 0 & \theta \\ 0 & 0_2 & 0 \\ 0 & \bar{\theta} & 0_4 \end{pmatrix} = \begin{pmatrix} 1_2 & \frac{1}{2}\theta\bar{\theta} & \theta \\ 0 & 1_2 & 0 \\ 0 & \bar{\theta} & 1_4 \end{pmatrix}; \tag{2.5}$$

[3] $g_L(\tau)$ has some restrictions on the fermionic parameters as explained in more detail in [16] and the section on the string below. Therefore $g_L(\tau)$ cannot remove all of the degrees of freedom from $g(\tau)$ since it contains fewer independent fermionic parameters than $\mathrm{SU}(2,2|4)_L$.

[4] The definition of L and J in this paper differ from the ones in [7,8] by overall factors. Consequently the formulas involving J in the present paper differ by the corresponding modifications from those of [7,8].

[5] It must be mentioned that, due to constraints, $\frac{1}{2}L^{MN}L_{MN}$ may not commute with g at the quantum level. Since this factor is sandwiched between g^{-1} and g, it must be passed through g before it is applied on Sp(2, R) invariant physical states. Only after this step, for $g \in$ SU(2, 2|4), one finds that the term involving $\frac{1}{2}L^{MN}L_{MN}$ vanishes on Sp(2, R) gauge invariant physical states. This result is verified by quantizing the system in fixed gauges, as seen below easily in the twistor gauge.

(3) partially fixing the $Sp(2, R)$ gauge symmetry by choosing the $M = +'$ doublet in the form $(X^{+'} = 1, P^{+'} = 0)$ and solving two of the constraints $X^2 = X \cdot P = 0$ to reduce the 2T phase space (X^M, P^M) to the gauge that describes the relativistic particle (x^μ, p^μ) in $d = 4$, namely, $X^{+'} = 1$, $X^{-'} = x^2/2$, $X^\mu = x_\mu$ and $P^{+'} = 0$, $P^{-'} = x.p$, $P^\mu = p^\mu$. Then the 2T system in Eq. (2.1) reduces to the 1T superparticle in Eq. (2.4) [15,16].

As is well known, the on shell quantum states of the superparticle described by Eq. (2.4) is given by the on shell fields of linearized $N = 4$ SYM. A quick way of understanding this is by performing quantization in the lightcone gauge, which gives 2^3 bosons and 2^3 fermions,[6], with on shell momenta in $3 + 1$ dimensions $|2^3_B, p> \oplus |2^3_F, p>$. These $8_{\mathrm{Bose}} + 8_{\mathrm{Fermi}}$ quantum states of the superparticle correspond to the transverse physical fields of $N = 4$, $d = 4$ SYM in the lightcone gauge with helicities (in parentheses) times their SU(4) multiplicities given by

$$(+1) \oplus (+1/2) \times 4 \oplus (0) \times 6 \oplus (-1/2) \times \bar{4} \oplus (-1). \tag{2.6}$$

Thus the $8_{\mathrm{Bose}} + 8_{\mathrm{Fermi}}$ states, taken in position space, correspond to the on shell SYM fields in the lightcone gauge that are classified by $SO(2) \subset SO(3, 1)$ as the little group that describes the helicities: $A_i(x)$ with $i = 1, 2$ for the SO(2) vector in transverse directions, $\psi^a_{1/2}(x)$, $\bar{\psi}_{-1/2,a}(x)$ for the SO(2) fermions in the 4 and $\bar{4}$ representations of SU(4) and $\phi^{[ab]}(x)$ for the SO(2) scalars in the six-dimensional antisymmetric tensor of SU(4). In this holographic picture, the original $SU(2, 2|4)_R$ global supersymmetry in Eq. (2.1) becomes the nonlinearly realized $N = 4$ superconformal symmetry, both of the gauge fixed action in Eq. (2.4) and of the $N = 4$ SYM action.

In 2T physics each gauge may appear to describe various 1T physics systems as holographic pictures in 3 + 1 dimensions, but the representation of the gauge invariant $SU(2, 2|4)_R$ cannot change by choosing some gauge since J is gauge invariant. Hence, it must be true that the rather complicated nonlinear representation of the superconformal supergroup $SU(2, 2|4)_R$ for the superparticle [15,20], properly operator ordered at the quantum level, must satisfy the projector condition $J^2 = J$. This is guaranteed by its 2T physics origin in the gauge invariant form $J = \frac{1}{4} g^{-1} L g$, which is then gauge fixed by inserting the gauge fixed versions of g, X, P given above [16].

Another gauge fixed form is the twistor description of the superparticle as discussed in [16]. This is done by using the $Sp(2, R)$ symmetry and the $SU(2, 2) \subset SU(2, 2|4)_L$ (that also locally rotates phase space X_i^M as SO(4,2); see string case below) to completely fix X^M, P^M to the form $X^{+'} = 1$ and $P^- = 1$ (note, not $P^{-'}$) while all other components vanish. These X^M, P^M already satisfy the constraints $X^2 = P^2 = X \cdot P = 0$. In this gauge the only nonvanishing component of L^{MN} is $L^{+'-} = 1$. Hence the 2T action in Eq. (2.1) and the $SU(2, 2|4)_R$ charges in Eq. (2.2) become

$$\mathcal{L}_{\mathrm{twistor}} = -\frac{1}{4} \mathrm{Str}(\partial_\tau g g^{-1} \Gamma) = \bar{Z}^A (\partial_\tau Z_A), \tag{2.7}$$

$$(J)_A^B = \left(\frac{1}{4} g^{-1} \Gamma g\right)_A^B = Z_A \bar{Z}^B. \qquad \Gamma \equiv \begin{pmatrix} \Gamma_{+'-} & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.8}$$

These twistor forms arise from one row of the matrix g and one column of the matrix g^{-1} since Γ has only one nonzero off-diagonal entry. It is evident from Eq. (2.7) that $Z_A, \bar{Z}^A$ are canonically conjugate complex supertwistors which can be expressed in terms of oscillators.[7] Because of their embedding in the supergroup element g, the supertwistors must satisfy $\bar{Z}^A Z_A = 0$, a condition which arises from an off-diagonal entry in $g g^{-1} = 1$. Furthermore, the condition $\bar{Z}^A Z_A = 0$ corresponds to $\mathrm{Str}(J) = Z_A \bar{Z}^A (-1)^A = \bar{Z}^A Z_A = 0$. The constraint $\bar{Z}^A Z_A = 0$ may also be interpreted as arising from a gauge symmetry U(1) of Eq. (2.1) as part of the original gauge symmetries in $SU(2, 2|4)_L$. Note the change of orders of operators in $Z_A \bar{Z}^A (-1)^A = \bar{Z}^A Z_A$ is valid at the quantum level without any constant residues in the case of $SU(n, n|2n)$.

The quantum states generated by the supertwistors are precisely the ones described by the well-known oscillator representation of the supergroup $SU(2, 2|4)_R$ [8,11] with the additional condition $\bar{Z}^A Z_A = 0$ that selects the physical states in super Fock space. These oscillator states

[6]After gauge fixing the remaining kappa supersymmetry, eight real components of the θ's remain. Upon quantization, they satisfy a Clifford algebra which is realized on 2^4 states, i.e., 2^3 bosons plus 2^3 fermions.

[7]We use the notation of [8] to identify the oscillators in the $SU(2) \times SU(2) \times SU(4)$ *unitary* basis of $SU(2, 2|4)$. They are

$$Z_A = \begin{pmatrix} a_n \\ \bar{b}^m \\ \psi_r \end{pmatrix},$$

$\bar{Z}^A = (\bar{a}^n, -b_m, \bar{\psi}^r)$ where a bar (such as $\bar{a}^n$) means creation operator, and otherwise annihilation operator. The extra minus sign in $\bar{Z}^A$ is due to the SU(2,2) metric in the $SU(2) \times SU(2)$ basis, and it is the reason for having an annihilation operator b for that entry instead of a creation operator (the canonical structure is imposed by the corresponding signs in the action). The indices take the values $n = 1, 2, m = 1, 2, \ r = 1, 2, 3, 4$. Then, after reordering the oscillators $\bar{Z}^A Z_A = \bar{a} \cdot a - (\bar{b} \cdot b + 2) + \bar{\psi} \cdot \psi$, we write the constraint $\bar{Z}^A Z_A = 0$ in terms of the number operators in the form $\Delta \equiv N_a - N_b + N_\psi = 2$. This is the $\Delta = 2$ condition (for one "color") in [8] that gave the SYM states as the supersingleton.

correspond precisely to the supersingleton representation of $SU(2,2|4)_R$ that describes $N = 4$ SYM. Again, the charges J in this gauge satisfy the projector condition $J^2 = J$ on physical states. This is easily verified directly in this gauge by using Eq. (2.8)

$$(J^2)_A^B = Z_A \bar{Z}^C Z_C \bar{Z}^B = Z_A \bar{Z}^B (\bar{Z}^C Z_C + 1) \underset{\text{phys. states}}{=} Z_A \bar{Z}^B = (J)_A^B. \tag{2.9}$$

In this computation we have used the oscillator commutation rules to pass the number operator $\bar{Z}^C Z_C$ through $\bar{Z}_B$ and then set $\bar{Z}^C Z_C = 0$ on physical states (kets), thus showing that the projector condition $J^2 = J$ is true on physical states.

One may choose other holographic pictures of the same 2T system, with varying physical interpretations of the 1T systems that arise in various gauges. For example, the $N = 4$ $AdS_2 \times S^2$ superparticle [$SO(1,2) \times SO(3)$ basis] and the $N = 4$ $AdS_3 \times S^1$ superparticle [$SO(2,2) \times SO(2)$ basis] emerge as duals to the supersymmetric particle [$SO(1,1) \times SO(3,1)$ basis] or the supertwistor system given above. Other examples of interest are the $N = 4$ supersymmetric hydrogen atom in three space dimensions [$SO(2) \times SO(4)$ or $SO(1,2) \times SO(3)$ bases] and the $N = 4$ harmonic oscillator in two space dimensions [$SO(2,2) \times SO(2)$ basis], which also emerge from gauge choices of Eq. (2.1). The purely bosonic versions of these examples (and some other generalizations) are discussed in detail in [6,19][8] at the classical and quantum levels. Each one of these $N = 4$ systems is represented by the $SU(2,2|4)_R$ supersingleton rearranged in various bases; hence each has a spectrum that is dual to the $N = 4$ SYM spectrum. The $SU(2,2|4)$ symmetry is interpreted as conformal symmetry in the superparticle gauge [$SO(1,1) \times SO(3,1)$ basis], but it has other interpretations as a nonlinear hidden symmetry in the other cases.

The interacting $N = 4$, $d = 4$ SYM theory, rewritten in the appropriate basis, may be taken as an interesting interacting theory for any of the 1T holographic pictures. There should also be a field theoretic formulation of this theory directly written covariantly in $4 + 2$ dimensions. The projector equation $J^2 = J$ is very suggestive as an equation of motion of cubic string field theory, and one may develop an interacting field theory along those lines for the 2T superparticle after introducing ghosts and a Becchi-Rouet-Stora-Tyutin (BRST) operator.[9].

III. 2T SUPERSTRING IN 4 + 2

We now present an action for a superstring in 2T physics in $4 + 2$ dimensions. This action has many holographic pictures in $3 + 1$ dimensions, with varying 1T physical interpretations, that parallel those of the 2T superparticle of the previous section. One of them is the twistor superstring.

The worldsheet "matter" fields are $X^M(\tau,\sigma)$, $[P^m(\tau,\sigma)]^M$, and the $SU(2,2|4)$ supergroup element $g(\tau,\sigma)$, which are the string analogs of the particle case, while the analogs of the three $Sp(2,R)$ gauge fields $A^{ij} = (A^{11}, A^{22}, A^{12})$ are now $[A(\tau,\sigma), B_{mn}(\tau,\sigma), C_m(\tau,\sigma)]$, respectively. The action is

$$\sqrt{-\gamma}\mathcal{L}^- = \partial_m X \cdot P^{-m} - \frac{1}{2} A X \cdot X - \frac{1}{2} B_{mn} P^{-m} \cdot P^{-n} - C_m P^{-m} \cdot X + \frac{1}{2} \mathrm{Str}(i\partial_m g g^{-1} L^{-m}) + \mathcal{L}_1^-, \tag{3.1}$$

with

$$L^{-m} \equiv \begin{pmatrix} \frac{i}{2}\Gamma^{MN} X_{[M} P_{N]}^{-m} & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.2}$$

Here $(P^{-m})^M$ is the chirally projected component of the worldsheet momentum current density $(P^{-m})^M = \frac{1}{2} \times (\sqrt{-\gamma}\gamma^{mn} - \varepsilon^{mn}) P_m^M$, where γ_{mn} is the worldsheet metric and ε^{mn} is the constant antisymmetric tensor. In what follows, it is important to realize that the projected P^{-m} has only one independent component on the worldsheet, since the opposite projector $(\sqrt{-\gamma}\gamma^{mn} + \varepsilon^{mn})/2$ annihilates it. $\mathcal{L}_1^- \equiv \mathcal{L}_1^-(A, B, C, \gamma, j^r)$ is an additional part of the action that contains the current $j_m^r(\tau,\sigma)$ of the twistor superstring [3,4] and perhaps the other fields. It will not be necessary to discuss details of $\mathcal{L}_1^-$ in this paper. Note that factors of $\sqrt{-\gamma}$ are already absorbed into the definition of the gauge fields (A, B_{mn}, C_m).

One may also introduce a Lagrangian $\mathcal{L}^+$ with the opposite worldsheet chirality projectors obtained from $\mathcal{L}^-$ by replacing $\varepsilon^{mn} \to -\varepsilon^{mn}$. It appears that one may formulate the twistor superstring either as an open string with both $\mathcal{L}^+ + \mathcal{L}^-$ and open string boundary conditions or as a closed string with only $\mathcal{L}^-$ [23]. We will concen-

[8] Another recent application of the 2T physics approach is the formulation of the adjoint representation of the high-spin algebra in terms of phase space (X^M, P^M) in any dimension [21]. As it should be expected, it corresponds to the $SO(d,2)$ singleton, whose quadratic Casimir $C_2 = 1 - d^2/4$ in any dimension was computed in [6]. The $SO(2) \times SO(d)$ basis described in [21] in terms of oscillators is an equivalent description of the phase space $SO(2) \times SO(d)$ basis of the H-atom gauge given in [19]. Applying the same methods, the work of [21] is generalized to the supersymmetric version of the high-spin algebra $hs(2,2|4)$ through our $SU(2,2|4)$ 2T system of Eqs. (2.1) and (2.2), either covariantly, or taken in a variety of gauges, all of which describe the supersingleton. The supersymmetric generalization for high-spin can be done also for the other dimensions discussed in this paper.

[9] See also the 2T physics field theory approaches along the lines of [22].

trate on the latter approach and hence study the properties of $\mathcal{L}^-$ in the rest of this paper.[10]

The action $S = \int d\tau d\sigma \sqrt{-\gamma}\mathcal{L}^-$ is clearly invariant under reparametrizations of the worldsheet. In the conformal gauge $\gamma_{mn} = \gamma\eta_{mn}$, it is convenient to choose coordinates $\sigma^\pm = \tau \pm \sigma$, and basis $\eta_{+-} = 1$, $\eta_{\pm\pm} = 0$, with $m, n = \pm$. The Lagrangian $\mathcal{L}^-$ contains only the $m = n = -$ components P^-, B_{--}, C_-, and takes the form

$$\sqrt{-\gamma}\mathcal{L}^- = \partial_- X \cdot P^- - \frac{1}{2}AX \cdot X - \frac{1}{2}B_{--}P^- \cdot P^- - C_- P^- \cdot X + \frac{1}{2}\mathrm{Str}(i\partial_- g g^{-1} L^-) + \mathcal{L}_1^-. \tag{3.3}$$

This looks just like the particle counterpart in Eq. (2.1) with $A^{11} \to A$, $A^{22} \to B_{--}$, $A^{12} \to C_-$, and $P \to P^-$ and therefore has the same structure of symmetries, but now local on the worldsheet instead of the worldline. Hence holographic pictures of the 4 + 2 dimensional 2T string are obtained in 3 + 1 dimensions by gauge choices just as in the particle case. One of these holographic pictures is the Witten-Berkovits twistor superstring.

Let us first discuss the symmetries in more detail in the case of the string in Eq. (3.1) before any gauge choices. There are three kinds of symmetries as itemized below.

(1) The $SU(2,2|4)_R$ global symmetry of the particle case is replaced by the transformation

$$g \to g' = g g_R, \qquad \partial_{-m} g_R = 0, \tag{3.4}$$

indicating that g_R is not a constant but is *holomorphic*. The fields $X, P^m, A, B_{mn}, C_n, \gamma_{mn}$ are neutral under this symmetry. The conserved Noether current for this symmetry is

$$J^{-m} = \frac{1}{4} g^{-1} L^{-m} g, \qquad \partial_m J^{-m} = 0. \tag{3.5}$$

The conservation is verified through the equations of motion. This current corresponds to a SU(2, 2|4) Kac-Moody algebra whose representations classify the physical states of the theory.

[10]A related purely bosonic string action, without the projectors $(\sqrt{-\gamma}\gamma^{mn} \pm \varepsilon^{mn})/2$, was considered in [17] in $d + 2$ dimensions. The conclusion in [17] was that the solution space of the 2T string reduced to a tensionless string in d dimensions after gauge fixing. Although the notation was slightly different, an action equivalent to the one in [17] is written in the present notation by dropping the projectors so that all components of P^m and all components of the gauge fields (A, B_{mn}, C_m) contribute. It turns out that the extra equation of motion $P^+ \cdot P^- = 0$ that would follow from varying B_{+-} is the one responsible for imposing tensionless strings for the solution space. With only $\mathcal{L}^-$ or $\mathcal{L}^+ + \mathcal{L}^-$ this condition is avoided since B_{+-} is absent. In any case, tensionless strings play a role in the overall scheme.

The three components of the $Sp(2, R)$ transformations in the particle case are replaced by the transformations $\delta_\alpha, \delta_\rho, \delta_\beta$, as follows:

(2a) Local dilatations δ_α:

$$\delta_\alpha X = \alpha X, \qquad \delta_\alpha P^m = -\alpha P^m, \qquad \delta_\alpha A = -2A\alpha, \qquad \delta_\alpha B_{mn} = 2B_{mn}\alpha, \qquad \delta_\alpha C_m = \partial_m \alpha. \tag{3.6}$$

The γ_{mn} and g fields are neutral under δ_α. Then we obtain $\delta_\alpha L^{-m} = 0$, $\delta_\alpha J^m = 0$, and $\delta_\alpha(\sqrt{-\gamma}\mathcal{L}^-) = 0$.

(2b) Local ρ-transformations δ_ρ:

$$\delta_\rho X = 0, \qquad \delta_\rho P^m = -\rho^m X, \qquad \delta_\rho A = 2C_m\rho^m + \partial_m \rho^m, \qquad \delta_\rho B_{mn} = 0, \qquad \delta_\rho C_m = B_{mn}\rho^n. \tag{3.7}$$

The γ_{mn} and g fields are neutral under δ_ρ. Then $\delta_\rho L^{-m} \sim \Gamma^{MN} X_{[M} X_{N]} \rho^{-m} = 0$, $\delta_\rho J^{-m} = 0$, while $\delta_\rho(\sqrt{-\gamma}\mathcal{L}^-) = -\frac{1}{2}\partial_m(\rho^{-m} X \cdot X)$ is a total derivative.

(2c) Local β-transformations δ_β:

$$\delta_\beta X = \beta_m P^{-m}, \qquad \delta_\beta P^m = 0, \qquad \delta_\beta A = 0, \qquad \delta_\beta B_{mn} = -C_{(m}\beta_{n)} + \frac{1}{2}\partial_{(m}\beta_{n)}, \qquad \delta_\beta C_m = -A\beta_m. \tag{3.8}$$

The γ_{mn} and g fields are neutral under δ_β. Then $\delta_\beta L^{-m} \sim \Gamma^{MN} P^{-m}_{[M} P^{-n}_{N]} \beta_n = 0$ [since the projected index $(-m)$ can take only one value]. This gives again $\delta_\rho J^{-m} = 0$, while $\delta_\beta(\sqrt{-\gamma}\mathcal{L}^-) = \partial_m(\frac{1}{2}\beta_n P^{-m} \cdot P^{-n})$ is a total derivative.

The local symmetries embedded in $SU(2,2|4)_L$ are as follows:

(3a) There is a local $SO(4,2) = SU(2,2)$ Lorentz symmetry with parameters $\varepsilon^{MN}(\tau, \sigma)$ under *left multiplication* of g in the spinor representation and simultaneous transformation of X^M, $(P^{-m})^M$ in the vector representation

$$\delta_\varepsilon X^M = \varepsilon^{MN} X_N, \qquad \delta_\varepsilon (P^{-m})^M = \varepsilon^{MN}(P^{-m}_M), \qquad \delta_\varepsilon g = \frac{1}{4}\varepsilon^{MN}\begin{pmatrix} \Gamma^{MN} & 0 \\ 0 & 0 \end{pmatrix} g. \tag{3.9}$$

The fields $A, B_{mn}, C_n, \gamma_{mn}$ are neutral under this symmetry. Then the current is invariant $\delta_\varepsilon J^{-m} = 0$. Furthermore, in the action, the derivatives $\partial_m \varepsilon^{MN}$ produced by the two kinetic terms in (3.1) cancel each other, while all other terms involving ε^{MN} also cancel, so that $\delta_\varepsilon(\sqrt{-\gamma}\mathcal{L}^-) = 0$. Similarly, there is an $SU(4) = SO(6)$ local symmetry with parameters $\varepsilon^{IJ}[(\tau, \sigma)]$ under *left multiplication* of g

$$\delta_\omega g = \frac{1}{4}\omega^{IJ}\begin{pmatrix} 0 & 0 \\ 0 & \Gamma_{IJ} \end{pmatrix} g. \tag{3.10}$$

The fields $X, P^m, A, B_{mn}, C_n, \gamma_{mn}$ are neutral under this symmetry. The derivative $\partial_m \varepsilon^{IJ}$ as well as other dependence on ε^{IJ} drops both in the current and in the action because Γ_{IJ} and Γ_{MN} appear in different blocks. (3b) Finally there is a local fermionic extended kappa (super)symmetry under *left multiplication* of g with infinitesimal fermionic coset elements $K\in SU(2,2|4)_L$ of the form

$$\delta_\kappa g = Kg, \qquad K = \begin{pmatrix} 0 & \xi \\ \tilde{\xi} & 0 \end{pmatrix}, \tag{3.11}$$

provided $\delta_\kappa A$, $\delta_\kappa B_{mn}$, $\delta_\kappa C_n$, are nonzero as specified below, and the local $\xi_s^a(\tau,\sigma)$ has the form

$$\xi_s^a = X^M(\Gamma_M \kappa_0^a)_s + (P^{-m})^M(\Gamma_M \kappa_m^a)_s, \tag{3.12}$$

with the local fermionic parameters $(\kappa_0)_s^a$ and $(\kappa_{-m})_s^a$ unrestricted.[11] The fields X, P^m, γ_{mn} are neutral under this symmetry. Then such a transformation gives for the current $\delta_\kappa J^{-m} = ig^{-1}[L^{-m}, K]g$, and for the action

$$\begin{aligned}\delta_\kappa(\sqrt{-\gamma}\mathcal{L}^-) = &-\frac{1}{2}\delta_\kappa A X\cdot X - \frac{1}{2}\delta_\kappa B_{mn}P^{-m}\cdot P^{-n} \\ &- \delta_\kappa C_m P^{-m}\cdot X \\ &+ \frac{1}{2}\mathrm{Str}(i\partial_m g g^{-1}[L^{-m}, K]),\end{aligned} \tag{3.13}$$

with

$$[L^{-m}, K] = X_{[M}P^{-m}_{N]}\begin{pmatrix} 0 & \Gamma^{MN}\xi \\ -\tilde{\xi}\Gamma^{MN} & 0 \end{pmatrix}. \tag{3.14}$$

Inserting the general form in (3.12) we examine the product

$$\begin{aligned}X_{[M}P^{-m}_{N]}(\Gamma_{MN}\xi) = X_{[M}P^{-m}_{N]}\Gamma_{MN}(X^R(\Gamma_R\kappa_0^a)_A + (P^{-m})^R \\ \times (\Gamma_R\kappa_m^a)_A).\end{aligned} \tag{3.15}$$

The three gamma term Γ_{MNR} in the gamma matrix algebra $\Gamma_{MN}\Gamma_R = \Gamma_{MNR} + \Gamma_M\eta_{NR} - \Gamma_N\eta_{MR}$ forces antisymmetry and drops out for any κ_0, κ_{-m}. The remaining one gamma terms give dot products $X\cdot X$, $P^{-m}\cdot P^{-n}$, $P^{-m}\cdot X$, and those terms can be cancelled in the action by the appropriate choice of $\delta_\kappa A$, $\delta_\kappa B_{mn}$, $\delta_\kappa C_n$. In the case of the current we obtain $\delta_\kappa J^{-m} = 0$ on physical states when the vanishing of the quantities $X\cdot X$, $P^{-m}\cdot P^{-n}$, $P^{-m}\cdot X$ are applied as constraints on physical states. Hence the action and the current are invariant under the local kappa supersymmetry embedded in $SU(2,2|4)_L$.

We can now specialize to some 1T cases of interest in $3+1$ dimensions by using the gauge symmetries to thin out the degrees of freedom from $4+2$ dimensions to those in $3+1$ dimensions. This gives various holographic pictures parallel to those discussed in the case of the particle in the previous section. One gauge produces a superstring in $3+1$ dimensions that parallels the superparticle case of Eq. (2.4). Another one is the twistor gauge that parallels Eq. (2.7), which we will discuss in more detail in order to establish the twistor superstring as a gauge fixed version of the 2T superstring.

We work in the conformal gauge $\sqrt{-\gamma}\gamma^{mn} = \eta^{mn}$ which reduces the system to the 2T string action in Eq. (3.3). Following the same arguments for the gauge choices that led to Eqs. (2.7) and (2.8) for the particle, we fix the gauge for the string and solve the constraints. This gives $X^M(\tau,\sigma) = (P^-)^M(\tau,\sigma) = 0$ for all τ,σ and all M except the following nonzero constant components for $M = +', -$

$$X^{+'}(\tau,\sigma) = 1, \qquad (P^-)^-(\tau,\sigma) = 1. \tag{3.16}$$

In this gauge the only nonzero component of $(L^-)^{MN}$ is $(L^-)^{+'-} = 1$, or $L^- = \Gamma$ as in Eq. (2.8), and therefore the 2T string action and its conserved $SU(2,2|4)_R$ current take the following gauge fixed forms

$$\mathcal{L}^- = -\frac{1}{4}\mathrm{Str}(\partial_- g g^{-1}\Gamma) + \mathcal{L}_1^- = \bar{Z}^A(\partial_- Z_A) + \mathcal{L}_1^-, \tag{3.17}$$

$$J_A{}^B = \left(\frac{1}{4}g^{-1}\Gamma g\right)_A^B = Z_A\bar{Z}^B, \tag{3.18}$$

where $Z_A(\tau,\sigma)$ are string supertwistors that satisfy the constraint

$$J_0 \equiv \bar{Z}^A Z_A = 0, \tag{3.19}$$

which should be applied on physical states. The constraint arises from the gauge symmetries of the 2T superstring as explained in the case of the superparticle in the previous section. The twistor system that has emerged in Eqs. (3.17), (3.18), and (3.19) is the same as the twistor superstring version suggested by Berkovits [3,4].

A new geometric description can also be given. As explained in [1] the geometric space described by the constrained twistors is $CP^{3|4}$. In the 2T physics approach we find that $CP^{3|4}$ is equivalent to the coset space

$$CP^{3|4} \leftrightarrow PSU(2,2|4)/H_\Gamma, \tag{3.20}$$

where H_Γ is the subgroup of $PSU(2,2|4)$ that commutes with the constant matrix Γ. This is the leftover gauge symmetry after fixing (X^M, P^M) as in Eq. (3.16). The H_Γ symmetry can remove further gauge degrees of freedom from g and reduce it to the constrained supertwistors, i.e., $CP^{3|4}$. The Lie superalgebra of H_Γ is embedded in a "triangular" configuration in the 8×8 supermatrix and is given by

$$h_\Gamma = su(1,1|4) + V_{(1,1|4)} + \bar{V}_{(1,1|4)} + R. \tag{3.21}$$

[11] However, since $X^2 = P^{+2} = X\cdot P^+ = 0$, the prefactors $X^M\Gamma_M$ and $P^{+mM}\Gamma_M$ have zero eigenvalues. Therefore only part of the kappa parameters can remove degrees of freedom from $g(\tau,\sigma)$ by gauge fixing [e.g., as in Eq. (2.5)]. So there remains physical fermionic degrees of freedom in g.

$V_{(1,1|4)}$ is in the fundamental representation of $su(1,1|4)$ and $\bar{V}_{(1,1|4)}$ is its complex conjugate. Some of the supercommutators are $[su(1,1|4), V_{(1,1|4)}\} \sim V_{(1,1|4)}$ and similarly for $\bar{V}_{(1,1|4)}$. Another of the nontrivial supercommutators is $[V_{(1,1|4)}, \bar{V}_{(1,1|4)}\} \sim R$, while R is an Abelian factor that commutes with all the other generators in h_Γ. The counting of *real* bosonic and fermionic parameters is

$$\text{PSU}(2,2|4)\text{:bosons} = 15 + 15, \quad \text{fermions} = 32; \tag{3.22}$$

$$su(1,1|4)\text{:bosons} = 3 + 1 + 15, \quad \text{fermions} = 16; \tag{3.23}$$

$$V_{(1,1|4)} + \bar{V}_{(1,1|4)}\text{:bosons} = 2 + 2, \quad \text{fermions} = 4 + 4; \tag{3.24}$$

$$R\text{:bosons} = 1. \tag{3.25}$$

From this we see that the coset $\text{PSU}(2,2|4)/\text{H}_\Gamma$ has six real bosons and eight real fermions, which is the correct number of real parameters in $\text{CP}^{3|4}$. This is also the correct number[12] of physical phase space degrees of freedom (x, p, θ) for the superparticle given in Eq. (2.4), as it should be. From this description of the geometry we see that we can present the supertwistor string as a gauged sigma model with the global group $\text{SU}(2,2|4)$ and the gauged subgroup H_Γ.

The arguments above show that the twistor superstring is a gauge fixed version of the 2T superstring. Hence the quantization of the 2T superstring in this gauge[13] can be performed by following the BRST quantization of the twistor system, including the appropriate ghosts, as in [3–5].

Here we make some additional remarks regarding the $\text{SU}(2,2|4)_R$ symmetry that classifies the physical states of the system. From the equations of motion it is evident that $\partial_- Z_A = 0$, so that Z_A and J_A^B are holomorphic as functions of $z = e^{i(\tau+\sigma)}$.

The quantization rules for the twistors may be summarized by the operator products

$$Z_A(z)\bar{Z}^B(w) \sim \frac{\delta_A{}^B}{z-w}. \tag{3.26}$$

Note that there is no pole in the sum $Z_A(z)\bar{Z}^A(w)(-1)^A = \bar{Z}^A(w)Z_A(z)$ due to the cancellation between an equal number of bosons and fermions. Hence there is no problem of singularities as $z \to w$ in imposing the constraint $J_0(z) \equiv \bar{Z}^A(z)Z_A(z) \sim 0$ on physical states at the quantum level.

The stress tensor is

$$T(z) = :\frac{1}{2}\partial_z\bar{Z}^A(z)Z_A(z) - \frac{1}{2}\bar{Z}^A(z)\partial_z Z_A(z): + t(z), \tag{3.27}$$

where $t(z)$ comes from the $\mathcal{L}_1^-$ part of the action. The dimensions of both Z_A and $\bar{Z}^A$ is $1/2$ since they are essentially Hermitian conjugates of each other except for the $\text{SU}(2,2|4)$ metric. Thus, as computed by using Eq. (3.26) we have

$$T(z)Z_A(w) \sim \frac{\frac{1}{2}Z_A(w)}{(z-w)^2} + \frac{\partial_w Z_A(w)}{(z-w)}, \tag{3.28}$$

and similarly for $\bar{Z}^A$. These dimensions are shifted from the dimensions given in [3–5] if we insist on Hermiticity with the spacetime signature for $\text{SO}(4,2) = \text{SU}(2,2)$. However, for the analytic continuation of SO(4,2) to the signature of $\text{SO}(3,3) = \text{SL}(4,R)$ and of $\text{SU}(2,2|4)$ to $\text{SL}(4|4;R)$ as used in [3–5], one may assign the dimensions $\dim(Z) = 0$ and $\dim(\bar{Z}) = 1$. This amounts to a shift in the stress tensor by a twist $T \to T_0(z) = T(z) - \frac{1}{2}\partial_z J_0 = -\bar{Z}^A(z)\partial_z Z_A(z): + t(z)$.

For the computation of various SYM helicity amplitudes in nontrivial instanton sectors one introduces further twisting to a stress tensor of degree n [3–5]

$$T_n(z) = T(z) - \frac{1}{2}(n+1)\partial_z J_0 = -\bar{Z}^A\partial_z Z_A - \frac{n}{2}\partial_z(\bar{Z}^A Z_A). \tag{3.29}$$

Relative to the twisted stress tensor the dimensions of Z_A and $\bar{Z}^A$ are $-n/2$ and $1 + n/2$, respectively,

$$\begin{aligned} T_n(z)Z_A(w) &\sim \frac{-\frac{n}{2}Z_A(w)}{(z-w)^2} + \frac{\partial_w Z_A(w)}{(z-w)}, \\ T_n(z)\bar{Z}^A(w) &\sim \frac{(1+\frac{n}{2})\bar{Z}^A(w)}{(z-w)^2} + \frac{\partial_w \bar{Z}^A(w)}{(z-w)}, \end{aligned} \tag{3.30}$$

as required in the SYM amplitude computations performed in [3–5].

By imposing the Virasoro and J_0 constraints on the physical states, i.e., requiring dimension one vertex operators for the degree zero stress tensor T_0 in Eq. (3.29) $T_0(z)V(w) \sim \frac{V(w)}{(z-w)^2} + \frac{\partial_w V(w)}{(z-w)}$, and $J_0(z)V(w) \sim 0$, one finds that only the states constructed with the lowest modes of $Z_A(z)$, $\bar{Z}_A(z)$, $j^a(z)$ satisfy the physical state conditions. Hence the twistor superstring is equivalent to a superparticle in the zero instanton sector. The only physical states are then the SYM supermultiplet (supersingleton, as in the previous section) in the adjoint representation of

[12] The physical phase space (x, p, θ) of the standard one-time superparticle, in d dimensions with N supersymmetries, is $2(d-1)$ bosons and N times half the dimension of the spinor representation (if the spinor is complex multiply with another factor of 2).

[13] A more complete BRST quantization would be to introduce the ghosts for all the gauge symmetries $\text{SL}(2,R) \times \text{SU}(2,2) \times \text{SU}(4) \times \text{Kappa}$ where $\text{SU}(2,2) \times \text{SU}(4) \times \text{Kappa} \subset \text{SU}(2,2|4)$. We postpone this to a later investigation.

the group G associated with the current j^r, plus the conformal supergravity multiplet (which contributes to loops). These are given by the dimension one vertex operators of the form $V_{\rm SYM}(z) = \phi_r[Z(z)]j^r(z)$ and $V_{\rm SUGRA}(z) = \bar{Z}^A(z)f_A[Z(z)] \oplus \partial Z_A(z)g^A[Z(z)]$, respectively, as explained in [3–5]. In computations of SYM helicity amplitudes, higher instanton sectors associated with T_n are needed; then the higher modes of the twistors contribute to those computations [3–5].

The $SU(2,2|4)_R$ current $J_A^B(z) = :Z_A(z)\bar{Z}^B(z):$ must be understood as being normal ordered at the quantum level. It follows from Eq. (3.26) that it satisfies the standard operator products between supercurrents

$$J_A{}^B(z)J_C{}^D(w) \sim \frac{\delta_A{}^D\delta_C{}^B(-1)^{BC+1}}{(z-w)^2} + \frac{(-1)^{BC}}{(z-w)} \times [-J_A{}^D(w)\delta_C{}^B + (-1)^{A(B+C)}J_C{}^B(w)\delta_A{}^D]. \quad (3.31)$$

By taking the supertrace and using $(-1)^A J_A{}^A = \bar{Z}^A Z_A = J_0$, we derive from the above the operator products

$$J_0(z)J_C{}^D(w) \sim -\frac{\delta_C{}^D}{(z-w)^2}, \qquad J_A{}^B(z)J_0(w) \sim -\frac{\delta_A{}^B}{(z-w)^2}, \qquad J_0(z)J_0(w) \sim 0, \quad (3.32)$$

without any single poles. After subtracting the part of $J_A{}^B(z)$ proportional to δ_A^B, the remaining $PSU(2,2|4)$ current has vanishing operator product with J_0. We also derive the matrix product of two $SU(2,2|4)$ currents, obtained by setting $B = C$ and summing. The coefficient of the double pole vanishes, and we remain with

$$J_A{}^B(z)J_B{}^D(w) \sim \frac{J_0(w)\delta_A{}^D}{(z-w)}. \quad (3.33)$$

Since $J_0(w) \sim 0$ on physical states, we see that the matrix product of operators $[J(z)J(w)]_A{}^D$ applied on physical states is finite as $z \to w$.

IV. 2T SUPERSTRING IN 10 + 2

The 2T superparticle in 4 + 2 dimensions discussed in a previous section was generalized to the 2T superparticle in 10 + 2 dimensions [7] by adding six more bosons (X^I, P^I), $I = 1, \cdots, 6$, that couple into the $SO(6) = SU(4)$ sector of $SU(2,2|4)$. The global supersymmetry does not change, it is still $SU(2,2|4)_R$, and it classifies the physical states. One of the gauge fixed versions of this 2T particle model gives the $AdS_5 \times S^5$ superparticle. Its quantum states were computed and summarized in Eq. (4.29) and footnote 4 of Ref. [7]. The spectrum matches to the well-known Kaluza-Klein towers of type-IIB supergravity compactified on $AdS_5 \times S^5$.

In this section we develop the parallel formalism for the 2T superstring in 10 + 2 dimensions. We will then choose an 1T gauge (equivalent to a left-moving $AdS_5 \times S^5$ string) that reduces the theory to a collection of eight supertwistors and their conjugates, subject to a set of constraints that satisfy the $S[U(2|2) \times U(2|2)]$ algebra. The constrained twistors describe the degrees of freedom in the coset space $SU(2,2|4)/S[U(2|2) \times U(2|2)]$. This space contains eight complex bosons and eight complex fermions which are equivalent to the physical phase space of $AdS_5 \times S^5$ string in a lightcone gauge. The well understood particle limit spectrum suggests an $SU(2,2|4)_R$ classification of string states in compactified 10 + 2 dimensions (F-theory or S-theory). It is found that this spectrum corresponds to the conserved high-spin currents expected in the weak coupling limit of $N = 4$, $d = 4$ SYM theory, in agreement with AdS-CFT correspondence.

The action of the 2T superstring in 10 + 2 dimensions has the same form as $\mathcal{L}^-$ in Eq. (3.1) but instead of six dimensions there are 12 dimensions labeled as $\hat{X}^{\hat{M}} = (X^M, X^I)$ and $\hat{P}^{-\hat{M}} = (P^{-M}, P^{-I})$, with $I = 1, \cdots, 6$, $M = +', -', \mu$, and $\mu = 0, 1, 2, 3$. The six dimensions labeled with M are the same as those of the 4 + 2 string, while the extra six dimensions labeled by I appear as their extension into 12 dimensions. Written in the conformal gauge as in Eq. (3.3) the Lagrangian takes the form

$$\sqrt{-\gamma}\hat{\mathcal{L}}^- = \partial_-\hat{X}\cdot\hat{P}^- - \frac{1}{2}A\hat{X}\cdot\hat{X} - \frac{1}{2}B_{--}\hat{P}^-\cdot\hat{P}^- - C_-\hat{P}^-\cdot\hat{X} + \frac{1}{2}\mathrm{Str}(i\partial_- gg^{-1}\hat{L}^-) + \mathcal{L}_1^-, \quad (4.1)$$

with

$$\hat{L}^- \equiv \begin{pmatrix} \frac{i}{2}\Gamma^{MN}X_{[M}P^-_{N]} & 0 \\ 0 & -\frac{i}{2}\Gamma^{IJ}X_{[I}P^-_{J]} \end{pmatrix}. \quad (4.2)$$

The extra dimensions appear in the 12-dimensional dot products $\hat{X}\cdot\hat{X}$, $\hat{P}^-\cdot\hat{P}^-$, $\hat{P}^-\cdot\hat{X}$ and in the second block of $\hat{L}^-$. The dot products are invariant under SO(10,2) but $\mathrm{Str}(i\partial_- gg^{-1}\hat{L}^-)$ reduces the symmetry to the subgroup $SO(4,2) \times SO(6)$. The extra minus sign in the lower block of $\hat{L}^-$ is cancelled by the extra minus sign in the supertrace.

The global and local symmetries are similar to those listed in Eqs. (3.4) to (3.15) with slight modifications that are outlined below. The most important modification is the kappa supersymmetry as discussed in item three below.

(1) The right side symmetry $SU(2,2|4)_R$ acts as in Eq. (3.4) and has a conserved holomorphic current $\hat{J}^- = \frac{1}{4}g^{-1}\hat{L}^- g$, just as before, but with $\hat{L}^-$ replacing L^-.

(2) The local $Sp(2,R)$ symmetry parameterized by α, β, ρ act on all 12 doublets $(\hat{X}^{\hat{M}}, \hat{P}^{-\hat{M}})$ and the gauge fields (A, B_{--}, C_-) as in Eqs. (3.6), (3.7), and (3.8). Both blocks of $\hat{L}^-$ are separately invariant

under Sp$(2,R)$. The SO(10,2) invariant Sp$(2,R)$ constraints $\hat{X}\cdot\hat{X}=\hat{P}^-\cdot\hat{P}^-=\hat{P}^-\cdot\hat{X}=0$ include all 12 dimensions, and not each six dimensional subset separately. Note that the solution space of these constraints include the case of vanishing extra dimensions $X_I=P_I^-=0$ (i.e. $4+2$ theory recovered as a special solution).

(3) The local SU$(2,2|4)_L$ symmetry acts on g as in Eqs. (3.9), (3.10), and (3.11), on (X^M,P^{-M}) as in Eq. (3.9), and on (X^I,P^{-I}) as $\delta_\omega X^I=\omega^{IJ}X_J$, $\delta_\omega(P^-)^J=\omega^{IJ}(P_J^-)$. Hence the SO$(4,2)=$ SU$(2,2)$ and SO$(6)=$ SU(4) are local symmetries. The kappa supersymmetry in Eqs. (3.12), (3.13), (3.14), and (3.15) is modified as follows. The form of ξ_s^a in K remains the same for the special solution subspace when the Sp$(2,R)$ constraints are satisfied with vanishing (X^I,P^{-I}), but otherwise is modified such that, instead of Eq. (3.12), we now have [still Sp$(2,R)$ invariant L^{-MI} or ξ_s^a]

$$\begin{aligned}\xi_s^a&=L^{-MI}(\Gamma_M\kappa\Gamma_I)_s^a\\&=X^M(\Gamma_M\kappa\Gamma_I)_s^aP^{-I}-P^{-M}(\Gamma_M\kappa\Gamma_I)_s^aX^I.\end{aligned}\quad(4.3)$$

There is only one free parameter κ_s^a instead of the two in Eq. (3.12). Then the δ_κ transformation of the current $\delta_\kappa J$ and the action $\delta_\kappa\hat{\mathcal{L}}^-$ produce terms involving $[\hat{L}^-,K]$ as in Eqs. (3.13) and (3.14), but $[\hat{L}^-,K]$ now has the form

$$[\hat{L}^-,K]\sim\begin{pmatrix}0 & L^{MN}(\Gamma_{MN}\xi)+(\xi\Gamma_{IJ})L^{IJ}\\ -L^{IJ}(\Gamma_{IJ}\bar{\xi})-(\bar{\xi}\Gamma_{MN})L^{MN} & 0\end{pmatrix}.\quad(4.4)$$

When the ξ in Eq. (4.3) is inserted in the structure $L^{MN}(\Gamma_{MN}\xi)+L^{IJ}(\xi\Gamma_{IJ})$, the three gamma terms Γ^{MNR} or Γ^{IJK} force antisymmetry, and drop out, while the remainder is seen to reduce to a linear combination of the SO(10,2) covariant dot products $\hat{X}\cdot\hat{X}$, $\hat{P}^-\cdot\hat{P}^-$, $\hat{P}^-\cdot\hat{X}$. Therefore, these can be cancelled in $\delta_\kappa\hat{\mathcal{L}}^-$ [see the form in Eq. (3.13)] by choosing $\delta_\kappa A$, $\delta_\kappa B_{--}$, $\delta_\kappa C_-$. Similarly, in the kappa variation of the current $\delta_\kappa J$ the Sp$(2,R)$ constraints vanish on physical states, so that $\delta_\kappa J\sim 0$ on Sp$(2,R)$ gauge invariant physical states. Hence there is a kappa supersymmetry in the $10+2$ theory embedded in SU$(2,2|4)_L$.

It should be noted that the kappa supersymmetry is smaller when the Sp$(2,R)$ constraints are satisfied with nonvanishing L^{IJ}. Then there is a single kappa parameter instead of two as noted above. This amount of kappa supersymmetry can remove only half of the fermions in $g(\tau,\sigma)$. By contrast, for the special solution for which the lower block of $\hat{L}^-$ vanishes, the larger kappa supersymmetry can remove 3/4 of the fermions in g as in the $4+2$ string of the previous section. The amount of kappa supersymmetry has a profound effect on the physical spectrum. With 1/2 kappa supersymmetry, as in the generic solutions of the $10+2$ theory, the physical spectrum (at the particle limit) is supergravity, while with 3/4 kappa supersymmetry, as in the $4+2$ theory or the equivalent special solution of the $10+2$ theory, the physical spectrum (at the particle limit) is SYM theory.

We now examine the physical content of this theory by choosing some 1T gauges. The $\mathrm{AdS}_5\times\mathrm{S}^5$ gauge is obtained by fixing two Sp$(2,R)$ gauges $(P^-)^{+'}(\tau,\sigma)=0$, $|X^I|(\tau,\sigma)=R=$ constant, and solving two of the Sp$(2,R)$ constraints $\hat{X}\cdot\hat{X}=\hat{X}\cdot\hat{P}^-=0$. The resulting phase space takes the form

$$\hat{M}=(+'\qquad -'\qquad \mu\qquad I),$$

$$\hat{X}^{\hat{M}}(\tau,\sigma)=\frac{R}{|\mathbf{y}|}\left(R,\quad\frac{x^2+\mathbf{y}^2}{2R},\quad x^\mu,\quad \mathbf{y}^I\right)(\tau,\sigma),\quad(4.5)$$

$$\hat{P}^{\hat{M}}(\tau,\sigma)=\frac{|\mathbf{y}|}{R}\left[0,\ \frac{1}{R}(x\cdot p+\mathbf{y}\cdot\mathbf{k}),\ \ p^\mu,\ \ \mathbf{k}^I\right](\tau,\sigma).\quad(4.6)$$

Evidently, we obtain $X^MX_M=-R^2$ and $X^IX_I=R^2$ which is the $\mathrm{AdS}_5\times\mathrm{S}^5$ space given by the metric

$$ds^2=d\hat{X}^{\hat{M}}d\hat{X}_{\hat{M}}=\frac{R^2}{y^2}[(dx^\mu)^2+(dy)^2]+(d\Omega)^2,\quad(4.7)$$

where $\Omega^I=\mathbf{y}^I/|\mathbf{y}|$ and $y=|\mathbf{y}|$. The boundary of the AdS_5 space at $y\to 0$ is Minkowski space x^μ in 4-dimensions. The SU$(2,2|4)_L$ gauge symmetry can be used to gauge fix g to the form Eq. (2.5). In this gauge the $10+2$ superstring reduces to a left-moving $\mathrm{AdS}_5\times\mathrm{S}^5$ superstring.

The particle limit of this theory was analyzed in this gauge, and its quantum spectrum was summarized in Eq. (4.29) and footnote 4 of Ref. [7]. The particle limit spectrum is 2^7 bosons and 2^7 fermions with $\mathrm{AdS}_5\times\mathrm{S}^5$ quantum numbers

$$\Phi_{2_B^7+2_F^7}(x^\mu,y,l)Y_l(\Omega),\quad(4.8)$$

where $Y_l(\Omega)$, $l=0,1,2,\cdots$ is a symbol for harmonics on S^5 [one row symmetric rank l traceless tensors of SO(6) constructed from the vector Ω^I]. These states satisfy the 12-dimensional mass shell condition $\hat{P}^2=0$, which in this gauge takes the form $\Delta_{\mathrm{AdS}_5}\Phi_{2_B^7+2_F^7}(x^\mu,y,l)=l(l+4)\Phi_{2_B^7+2_F^7}(x^\mu,y,l)$. The $l=0$ case is the special solution that reduces to the $4+2$ superparticle, which has a larger kappa symmetry. Therefore, for $l=0$ the spectrum reduces to the short supermultiplet with 2^3 bosons plus 2^3 fermions, which gives the SYM supermultiplet, as already discussed earlier in this paper. For general l, since the model has a global symmetry SU$(2,2|4)_R$ the states are classified as towers of SU(2,2) distinguished by the SU$(4)=$ SO(6) quantum number l. It was shown that for $l\geq 1$ this is the same as the Kaluza-Klein spectrum of linearized type-IIB supergravity compactified on

$\mathrm{AdS}_5 \times \mathrm{S}^5$ while for $l = 0$ it is the singleton equivalent to the $N = 4$, $d = 4$ SYM spectrum, as discussed earlier in this paper.

The results of the particle case described in the previous paragraph suggest that the string case of the present paper generalizes the compactified type-IIB supergravity spectrum to a compactified string spectrum on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and furthermore that this spectrum should be organized as representations of the current algebra $SU(2,2|4)_R$ since this is the global symmetry of the theory. There remains to study the representations of this noncompact super current algebra or use related methods (such as supertwistors as described below) to study the spectrum and further properties of the theory.

Before plunging into detailed computation it is interesting to note that there is a candidate spectrum that was suggested in 1995 on the basis of symmetries in M-theory [9] and was recently revived in the context of $N = 4$, $d = 4$ SYM theory and the AdS-CFT correspondence [10]. This provides a useful guide to organize the spectrum we are seeking, to relate it to other interesting concepts, and to simultaneously use the newly emerging framework as a basis for the group theoretical classification found in [9,10].

In 1995 it was suggested that the massive 10D type-IIA string spectrum could be extended to compactified 11D M-theory massive spectrum, including Kaluza-Klein (KK) states, just like the massless 10D type-IIA spectrum is extended to compactified 11D supergravity spectrum. The guiding tool was the little group SO(10) for massive states in 11D, and one needed to find the completion of the SO(9) massive string spectrum into SO(10) representations $SO(9) \subset SO(10) \subset SO(10,1)$. In this way a systematic formula for the spectrum including Kaluza-Klein states was discovered at all string levels. The formula given in Eq. (3.8) in [9] is very simple. Define the total level n as the string level $n - k$ plus the KK level k. Start with the left-moving states at string mass level n at KK level 0, then add the left-moving string states of level $n - 1$ at KK level 1, plus those of string level $n - 2$ at KK level 2, and so on, up to the left-moving string states of level 1 at KK level $n - 1$. Repeat the same procedure for the right-moving sector at total level n, and then take the product of left $\times$ right movers each with total level n. The collection of these states form SO(10) multiplets for every total level $n \geq 1$. The SO(10) representations obtained in this way were given explicitly up to total level $n = 5$ in [9].

To apply this formula to the present case, recall that the T-dual of type-IIA is type-IIB. As long as one discusses the little group $SO(9) \subset SO(9,1)$ of the string, there is no difference between starting with SO(9) representations of type-IIA or type-IIB strings. The higher dimensional extension of type-IIB is F-theory [12] or S-theory [13,14] in $10 + 2$ dimensions, with the compact subgroup SO(10). The SO(10) classification of states described in the previous paragraph can be interpreted (via T-duality) as those of a $10 + 2$ dimensional theory. The $10 + 2$ superstring suggested in this paper is expected to have a closely related spectrum after compactification of $10 + 2$ to $(4 + 2) + (6 + 0)$, with $SO(10) \rightarrow SO(4) \times SO(6)$. Indeed, we have already argued in this section that the $10 + 2$ string can be viewed as a 1T string on an $\mathrm{AdS}_5 \times \mathrm{S}^5$ background, and that its particle limit produces the compactified type-IIB supergravity spectrum. Therefore, to compare the spectrum of [9] to the present case, the SO(10) multiplets given in [9] should be decomposed into $SO(4) \times SO(6)$. Furthermore the SO(6) quantum numbers coming from the harmonic expansion of higher dimensional fields into Kaluza-Klein towers should be included, as in Eq. (4.8). These towers should then produce a series of $SU(2,2|4)$ representations that can be compared to the $SU(2,2|4)_R$ current algebra spectrum we are seeking.

Based on AdS-CFT correspondence we might expect that the infinite $N = 4$, $d = 4$ SYM theory has a close relationship with the spectrum produced by our $10 + 2$ string taken in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ gauge described above. In particular, we already know that the $10 + 2$ string has the special solution sector of the $4 + 2$ string, which is indeed related to $N = 4$, $d = 4$ SYM theory through the twistor superstring, as shown earlier in this paper. As further evidence we note that the $SO(10) \rightarrow SO(4) \times SO(6)$ reduction process described in the previous paragraph has precisely the same content of towers of $SO(4) \times SO(6)$ representations as the classification of high-spin currents [10] expected in the weak coupling limit [24] of $N = 4$, $d = 4$ SYM theory. This is encouraging for our expected results on the spectrum of the $10 + 2$ string. The work in [10] parallels the group theoretical steps in [9], while the current paper provides a dynamical string model with the same group theoretical properties and with connections to SYM. The conclusive analysis of the dynamics and of the group theory could be achieved through the twistor framework emerging in the current paper, as described in the remainder of this section.

To analyze the $10 + 2$ superstring we now choose a twistor gauge instead of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ gauge of Eqs. (4.5) and (4.6). This is the analog of the twistor gauge of Eq. (2.7) instead of the particle gauge of Eq. (2.4) for the superparticle. Thus, we first use the local $SO(4,2) \times SO(6) \subset SU(2,2|4)_L$ to rotate the 12 components of $\hat{X}^{\hat{M}}(\tau, \sigma)$ so that they point in the special directions $M = 0'$ and $I = 1$, and also impose the constraint $\hat{X} \cdot \hat{X} = 0$. The result is the 12-dimensional lightlike vector $\hat{X}^{\hat{M}} \sim (1,0,0,0,0,0;1,0,0,0,0,0)$, assuming that we are analyzing the L^{IJ} nonzero sector of the theory (i.e., not the $4 + 2$ special solution, which is already discussed in the previous section). There still remains local symmetry $SO(4,1) \times SO(5) \subset SO(4,2) \times SO(6)$ which does not change the gauged fixed form of $\hat{X}$. Using this we can rotate P^{-M} and P^{-I} to special directions with at least four zero components each. Then, using the $\mathrm{Sp}(2,R)$ local symmetry some of the remaining nonzero components can be rotated to zero. Finally, applying the remaining constraints $\hat{X}\cdot\hat{P}^{-}=\hat{P}^{-}\cdot\hat{P}^{-}=0$ we can complete the gauge fixing of the 12 dimensional phase space $\hat{X}^{\hat{M}}$, $\hat{P}^{-\hat{M}}$ to the form of two lightlike orthogonal vectors in 12 dimensions

$$\hat{M}=(0'\ 0\ 1\ \cdots\ 4\ ,I=1\ 2\ 3\ \cdots\ 6),$$

$$\hat{X}^{\hat{M}}(\tau,\sigma)\sim(1\ 0\ 0\ \cdots\ 0\ ,1\ 0\ 0\ \cdots\ 0), \tag{4.9}$$

$$\hat{P}^{\hat{M}}(\tau,\sigma)\sim(0\ 1\ 0\ \cdots\ 0\ ,0\ 1\ 0\ \cdots\ 0). \tag{4.10}$$

In this gauge the 8×8 matrix $\hat{L}^{-}$ simplifies to

$$\hat{L}^{-}\sim\begin{pmatrix} i\Gamma_{0'0} & 0\\ 0 & i\Gamma_{12}\end{pmatrix}\equiv\hat{\Gamma}=\begin{pmatrix}1_2 & 0 & 0 & 0\\ 0 & -1_2 & 0 & 0\\ 0 & 0 & -1_2 & 0\\ 0 & 0 & 0 & 1_2\end{pmatrix}, \tag{4.11}$$

since the only nonzero components of L^{MN}, L^{IJ} are $L^{0'0}$, L^{12}, respectively, and furthermore $L^{0'0}=L^{12}$. We chose a particular basis of gamma matrices so that $i\Gamma_{0'0}$ and $i\Gamma_{12}$ are diagonal as written. Note that $\hat{\Gamma}$ is an invariant under $\mathrm{H}_{\hat{\Gamma}}=\mathrm{S}[\mathrm{U}(2|2)\times\mathrm{U}(2|2)]$ transformations embedded in $\mathrm{SU}(2,2|4)$. Therefore, $\mathrm{H}_{\hat{\Gamma}}\subset SU(2,2|4)_L$ is a remaining local symmetry that can remove further degrees of freedom from the group element $g(\tau,\sigma)$. The first $\mathrm{SU}(2|2)$ acts on rows $R=1,2,7,8$ (labeled as r below) and the second $\mathrm{SU}(2|2)$ acts on rows $R=3,4,5,6$ (labeled as r' below) as seen from the form of $\hat{\Gamma}$. The remaining U(1) has a generator proportional to $\hat{\Gamma}$.

In this gauge the action in Eq. (4.1) and the $\mathrm{SU}(2,2|4)_R$ symmetry current reduce to

$$\begin{aligned}\sqrt{-\gamma}\hat{\mathcal{L}}^{-}&=\frac{1}{4}\mathrm{Str}(\partial_{-}gg^{-1}\hat{\Gamma})+\mathcal{L}_1^{-}\\ &=\bar{Z}_r^A\partial_{-}Z_A^r(-1)^r-\bar{Z}_{r'}^A\partial_{-}Z_A^{r'}(-1)^{r'}+\mathcal{L}_1^{-},\end{aligned} \tag{4.12}$$

$$(\hat{J}^{-})_A{}^B=\left(\frac{1}{4}g^{-1}\hat{\Gamma}g\right)_A^{\ \ B}=Z_A^r\bar{Z}_r^B-Z_A^{r'}\bar{Z}_{r'}^B. \tag{4.13}$$

Thus, the theory is described by a collection of eight supertwistors and their conjugates, Z_A^r, $\bar{Z}_r{}^A$, $r=1,2,7,8$, and $Z_A^{r'}$,$\bar{Z}_{r'}{}^A$, $r'=3,4,5,6$. The twistors labeled by $r=1,2$ and $r'=3,4$ have bosons in their first four components $A=1,2,3,4$ [basis for $\mathrm{SU}(2,2)\subset\mathrm{SU}(2,2|4)_R$] and fermions in their last four components $A=5,6,7,8$ [basis for $\mathrm{SU}(4)\subset\mathrm{SU}(2,2|4)_R$]. By contrast, the twistors labeled by $r=7,8$ and $r'=5,6$ are unusual since they have fermions in the SU(2,2) basis and bosons in the SU(4) basis. This structure is dictated by the fact that, combined together, they make up the group element g. The raising or lowering of the indices on the twistors and their conjugates is done in accordance with the fact that g^{-1} is constructed by taking the Hermitian conjugate, and multiplying with the $\mathrm{SU}(2,2|4)$ metric, $g^{-1}=\eta g^{\dagger}\eta$. Therefore, the twistors are constrained by the condition $g^{-1}g=1$, which requires

$$\begin{gathered}(j)_{r_1}{}^{r_2}\equiv\bar{Z}_{r_1}{}^AZ_A{}^{r_2}=\delta_{r_1}^{r_2},\qquad (j')_{r_1'}{}^{r_2'}\equiv\bar{Z}_{r_1'}{}^AZ_A{}^{r_2'}=\delta_{r_1'}^{r_2'},\\ \bar{Z}_r{}^AZ_A{}^{r'}=\bar{Z}_{r'}{}^AZ_A{}^r=0.\end{gathered} \tag{4.14}$$

These may be understood as arising from the remaining gauge symmetry

$$\mathrm{H}_{\hat{\Gamma}}=\mathrm{S}[\mathrm{U}(2|2)\times\mathrm{U}(2|2)]_L\subset\mathrm{SU}(2,2|4)_L. \tag{4.15}$$

These constraints also guarantee that the$\mathrm{SU}(2,2|4)_R$ current $(\hat{J}^{-})_A{}^B$ has zero supertrace

$$\begin{aligned}\mathrm{S\,tr}(\hat{J}^{-})&=(Z_A^r\bar{Z}_r^A-Z_A^{r'}\bar{Z}_{r'}^A)(-1)^A\\ &=\bar{Z}_r^AZ_A^r(-1)^r-\bar{Z}_{r'}^AZ_A^{r'}(-1)^{r'}\end{aligned} \tag{4.16}$$

$$=\mathrm{Str}(j)-\mathrm{Str}(j')=\mathrm{Str}(1)-\mathrm{Str}(1)=0, \tag{4.17}$$

where the patterns of signs $(-1)^r$, $(-1)^{r'}$, $(-1)^A$ take into account the interchange of orders of bosons and fermions and the definition of supertrace.

The quantization of this twistor system is given by the operator products

$$\begin{gathered}Z_A^{r_1}(z)\bar{Z}_{r_2}^B(w)\sim\frac{\delta_A^B\delta_{r_2}^{r_1}}{z-w},\qquad Z_A^{r_1'}(z)\bar{Z}_{r_2'}^B(w)\sim-\frac{\delta_A^B\delta_{r_2'}^{r_1'}}{z-w},\\ Z_A^r(z)\bar{Z}_{r'}^B(w)\sim 0.\end{gathered} \tag{4.18}$$

Equivalently, we can write for the group element

$$(g^{-1})_A^{R_1}(z)\ (g)_{R_2}^B(w)\sim\frac{\delta_A^B\Gamma^{R_1}_{R_2}}{z-w}, \tag{4.19}$$

with the current $(\hat{J}^{-})_A{}^B=(\frac{1}{4}g^{-1}\Gamma g)_A{}^B$ and the constraints as given above.

These quantization rules are equivalent to a system of bosonic and fermionic oscillators which are constrained as indicated. From the point of view of the oscillator formalism for noncompact supergroups [8,11] the system can be interpreted as oscillators with a "color" supergroup $\mathrm{S}[\mathrm{U}(2|2)\times\mathrm{U}(2|2)]_L$. The physical states are the "color" singlets. Thus, the representation space of the physical currents $\mathrm{SU}(2,2|4)_R$ can be analyzed with the kinds of algebraic oscillator methods used in the past, after taking into account the fact that the "color" group is now a supergroup and restricting the Fock space states to the "color" singlet sector as in [8].

It should also be noted that geometric methods based on the coset

$$SU(2,2|4)/S[U(2|2)\times U(2|2)], \tag{4.20}$$

could provide another useful approach for analyzing the theory. One may introduce the gauge fields of $S[U(2|2)\times U(2|2)]$ explicitly to present the twistor model above as a gauged sigma model. This coset contains two real bosons plus eight complex bosons, and eight complex fermions (i.e., 18 real bosons and 16 real fermions). This counting of independent degrees of freedom is the same as the super phase space (positions, momenta and fermions) of the $AdS_5\times S^5$ string of Eqs. (4.5) and (4.6) after fixing a physical gauge. This makes it evident that the constrained twistor space given above is equivalent to the conventional description in usual spacetime, as expected from the fact that they are both obtained by gauge fixing the same $10+2$ superstring. In particular, the fermionic zero modes of this coset create 2^7 bosons and 2^7 fermions. Given the $SU(2,2|4)$ symmetry of the model, it is evident that these are the correct states that describe the compactified supergravity multiplet, as expected from Eq. (4.8).

The technical analysis of the twistor system above is incomplete at this stage. We hope to discuss it in a future paper.

V. FURTHER REMARKS ON OTHER DIMENSIONS

The 2T superstring in $4+2$ dimensions is directly generalized to $d+2$ dimensions for the special dimensions $d=3,4,5,6$ by taking $(X^M,P^{mM})(\tau,\sigma)$ in the corresponding $d+2$ dimensions and using the supergroup element $g(\tau,\sigma)\in G$, with G given by $OSp(8|4)$, $SU(2,2|4)$, $F(4)$, $OSp(8^*|4)$, respectively, for $d=3,4,5,6$. The 2T Lagrangian has the same form as (3.1), except for modifying L, L^{-m} in Eqs. (2.1) and (3.2) by replacing $\frac{i}{2}\Gamma_{MN}L^{MN}\rightarrow i\frac{2}{s}\Gamma_{MN}L^{MN}$, where s is the dimension of the spinor in $d+2$ dimensions. The conserved current is as before $J=\frac{1}{2}g^{-1}Lg$ for the group G, and the local symmetries are exact parallels as the ones discussed in items 1, 2a, 2b, 2c, 3a, 3b in Sec. III. The normalization $i\frac{2}{s}\Gamma_{MN}L^{MN}$ is needed to insure the $SO(d,2)$ local symmetry in item 3a.

Just like the $d=4$ case, in the particle limit in each one of these string models for $d=3,4,5,6$, the physical states consist of eight bosons and eight fermions. To understand this consider the particle limit of the $3+2$, $4+2$, $5+2$, and $6+2$ models which was discussed in [15,16]. When the relativistic particle type gauge is chosen, the resulting superparticle is described by the Lagrangian in Eq. (2.4) taken for the corresponding dimension d and the corresponding number N of supersymmetries determined by G. In each case, $g(\tau,\sigma)$ is such that it contains 32 real fermionic degrees of freedom $\Theta_s^a(\tau,\sigma)$, but the local kappa supersymmetry of Eq. (3.12) removes half of them so that θ_α^a in the relativistic superparticle gauge of Eq. (2.4) contains 16 real fermionic degrees of freedom. The remaining kappa supersymmetry of the superparticle removes half of what is left, so that the *physical fermionic zero modes is eight* for each of the $3+2$, $4+2$, $5+2$, $6+2$ models. When the superparticle is quantized in the lightcone gauge, these eight fermionic zero modes create 2^3 bosonic physical states and 2^3 fermionic physical states, for each of the models given in the first paragraph of this section. These states are then classified with the little group $SO(d-2)$ in the lightcone gauge and with the R symmetry group contained in the supergroup G.

For $d=3,4$ we find that these superparticle quantum states are in one to one correspondence with the physical fields of SYM theory with $N=8,4$ supersymmetries in $d=3,4$ dimensions taken the lightcone gauge (eight bosons and eight fermions). There is a quick way of understanting this result. The compactification of the $d=10$ superparticle (with its 16 fermionic degrees of freedom) to $d=3,4$ dimensions gives the superparticle of Eq. (2.4) with the correct number of supersymmetries $N=8,4$, respectively, that match those of the gauge fixed 2T superparticle. Thus the quantum states of the superparticle in Eq. (2.4) must coincide with the compactification of the physical quantum states of the $d=10$ superparticle, which is $8_{\rm vector}+8_{\rm spinor}$ of the little group $SO(8)$ in $SO(9,1)$. When these $SO(8)$ representations are reduced to the little groups Z_2, $SO(2)$ for $d=3,4$, respectively, they describe the physical degrees of freedom of the superparticle as well as of SYM in $d=3,4$. Recall that the quantum states of the $d=10$ superparticle correspond to the eight bosonic and eight fermionic fields of the ten dimensional SYM theory taken in the lightcone gauge. Hence the quantum states created by the zero modes of the $3+2$, $4+2$ string models precisely correspond to the quantum fields of the SYM theory in the dimensions $d=3,4$, respectively.

Similarly, for $d=6$ the quantum states of the superparticle are related to the fields of a special superconformal field theory that contains an antisymmetric tensor $B_{\mu\nu}$ with self dual field strength $H_{\mu\nu\lambda}=\partial_{[\lambda}B_{\mu\nu]}=H^*_{\mu\nu\lambda}$, five scalars ϕ^i and fermions ψ_α^a, with a, i indicating the spinor and vector of the $Sp(4)$ R-symmetry. In the lightcone gauge of this field theory, the transverse degrees of freedom B_{mn} describe a self dual antisymmetric tensor of the transverse $SO(4)=SU(2)\times SU(2)$. Therefore it has 3 independent degrees of freedom classified as $(j_1,j_2)=(1,0)$. These together with the five scalars ϕ^i correspond to the eight bosons, while ψ_α^a supplies also 8 physical fermionic degrees of freedom in the lightcone gauge. These are precisely the eight bosons and eight fermions produced by the superparticle of Eq. (2.4) as follows: the gauge fixing of the superparticle all the way to the lightcone gauge removes 3/4 of the original 32 fermions Θ_s^a of the 2T superparticle, leaving behind eight zero mode $\theta' s$ that are classified as $(\frac{1}{2}, 0; 4)$ under the little group $SO(4) \times Sp(4) \subset SO(8^*|4)$, where the $SO(4) = SU(2) \times SU(2)$ representation is given as $(j_1, j_2) = (\frac{1}{2}, 0)$. These zero modes consist of four creation and four annihilation operators. When applied on the vacuum they create eight bosons classified as $(1, 0; 0) + (0, 0; 5)$ and eight fermions classified as $(\frac{1}{2}, 0; 4)$ under $SO(4) \times Sp(4)$. These match the transverse lightcone fields of the $d = 6$ superconformal theory as described above. This special theory is believed to be interacting and conformal at the quantum level [25,26] but it has been difficult to study it because of the lack of a covariant field theoretic action. The twistor superstring formalism description given below in this paper could be a possible approach for studying this theory in the same way as the Witten-Berkovits twistor superstring is used to analyze SYM theory.

The results described in the previous paragraphs give the physical spectrum of the 2T superstring theories with the conserved current $J^- = \frac{1}{4} g^{-1} \hat{L}^- g$ for the superconformal groups G given above. What are the unitary representations of G that emerge, and is there a sigma model type geometrical description of these models? To answer these questions we investigate the twistor gauge since the same physical content of the 2T theory can be recovered in any gauge. The result for $d = 4$ is already discussed in the other sections of this paper, while for $d = 3, 5, 6$ it is summarized as follows:

(i) For $d = 3$ the twistor Z_A is real, it contains eight fermions and four bosons and is in the fundamental representation of $OSp(8|4)$. At the classical level it satisfies the condition $\bar{Z}^A Z_A = 0$ automatically without constraining the degrees of freedom [$OSp(8|4)$ metric is one in the fermi sector and is antisymmetric in the bose sector]. At the quantum level it contains two bosonic oscillators and their conjugates [classified by $SU(2) \times U(1) \subset Sp(4)$] and four fermionic oscillators and their conjugates [classified by $SU(4) \times U(1) \subset SO(8)$]. The resulting oscillator representation in Fock space is the supersingleton of $OSp(8|4)$ and this describes $d = 3$, $N = 8$ SYM spectrum as expected from the discussion above. In this gauge the current becomes $J_A^B = (\frac{1}{4} g^{-1} \Gamma g)_A^B = Z_A \bar{Z}^B$, and there is a triangular subgroup $H_\Gamma \subset OSp(8|4)$ that commutes with the constant Γ, whose algebra is $h_\Gamma = osp(8|2) + V_{8|2} + R$, such that $[osp(8|2), V_{8|2}\} \sim V_{8|2}$ and $[V_{8|2}, V_{8|2}\} \sim R$, while the Abelian factor R commutes with all the generators in h_Γ. The coset space $OSp(8|4)/H_\Gamma$ has the correct counting of parameters[14] that corresponds to the twistors; these describe the geometric space. As expected, this has the same number of phase space degrees of freedom (x, p, θ) as the $d = 3$, $N = 8$ superparticle (see footnote [12]).

(ii) For $d = 6$ the matrix Γ has two nonzer entries instead of one. Therefore there are two $OSp(8^*|4)$ twistors Z_A^i, with $i = 1, 2$. These twistors each contain eight bosons and four fermions (opposite of the $d = 3$ case) and are constrained as a traceless tensor $\bar{Z}_i^A Z_A^j - \frac{1}{2} \delta_i^j \bar{Z}_k^A Z_A^k = 0$. These three bosonic constraints form an sl(2) local symmetry, therefore together with the gauge fixing of sl(2) they remove $3 + 3 = 6$ bosonic degrees of freedom from the Z_A^i. Hence the number of unconstrained bosons in the geometric space described by the twistors is $2 \times 8 - 6 = 10$, while the number of fermions is $2 \times 4 = 8$. This is the same number as the physical phase space degrees of freedom (x, p, θ) for $d = 6$, $N = 2$ superparticle (see footnote [12]). The coset description is found by noting that the triangular subalgebra that commutes with Γ is $h_\Gamma = [osp(2, 2|4) + sl(2)] + V_{(4|4),2} + R$, such that $[(osp(2, 2|4) + sl(2)), V_{(4|4),2}\} \sim V_{(4|4),2}$ and $[V_{(4|4),2}, V_{(4|4),2}\} \sim R$ while the Abelian factor R commutes with all the generators in h_Γ. Thus, the geometric space is $OSp(8^*|4)/H_\Gamma$ with ten bosons and eight fermions, which is the expected number.[15] The quantized constrained twistors generate the oscillator representation of the noncompact superalgebra $OSp(8^*|4)$ with the "color" group sl(2) acting on the $i = 1, 2$ index. Only the sl(2) "color" singlet states are kept in the Fock space as the physical states. The resulting representation is the doubleton of $OSp(8^*|4)$ and this indeed describes the physical fields of the $d = 6$, $N = 2$, superconformal theory as expected from the previous discussion above.

(iii) For $d = 5$ we expect a geometric space $F(4)/H_\Gamma$ whose dimension is eight bosons and eight fermions, since this is the counting for the physical phase space degrees of freedom (x, p, θ) for $d = 5$, $N = 2$ superparticle (see footnote 12). F(4) has 24 bosons [$SO(5, 2) \times SO(3)$ subgroup] and 32 real fermions in the complex (8,2) spinor rep-

[14] $OSp(8|4)$ has $28 + 10$ bosons and $4 \times 8 = 32$ fermions. $OSp(8|2)$ has $28 + 3$ bosons and $2 \times 8 = 16$ fermions. $V_{8|2}$ is classified as the fundamental representation of $OSp(8|2)$ with eight fermions and two bosons.

[15] $osp(8^*|4)$ has $28 + 10$ bosons and $4 \times 8 = 32$ fermions. $osp(2, 2|4) + sl(2)$ has $6 + 10 + 3$ bosons and $4 \times 4 = 16$ fermions. $V_{(4|4),2}$ is classified as the fundamental representation of $osp(4|4)$ and a doublet of sl(2). with $4 \times 2 = 8$ fermions and $4 \times 2 = 8$ bosons.

resentation of $SO(5,2) \times SO(3)$. Therefore the triangular subgroup H_Γ must contain 16 bosons and 24 fermions[16]. Its algebra is $h_\Gamma = [psu(2|2) + sl(2)] + V_{(3|8),2} + R$, such that $[(psu(2|2) + sl(2)), V_{(3|8),2}\} \sim V_{(3|8),2}$ and $[V_{(3|8),2}, V_{(3|8),2}\} \sim R$ while the Abelian factor R commutes with all the generators in h_Γ. The coset space $F(4)/H_\Gamma$ should describe a conformal theory in $d = 5$ and $N = 2$ as expected from the superparticle spectrum discussed above. It is harder to describe the space in terms of twistors because F(4) does not have an $8|2$ dimensional fundamental representation which would have corresponded to the spinor space of $SO(5,2) \times SO(3)$. For the same reason the oscillator representation has not been developed.

From the discussion above we see that each of these models can be presented geometrically as a gauged sigma model based on the global group G and gauged with the subgroup H_Γ.

The extension from $d + 2$ to higher dimensions $d + d' + 2$, with the addition of extra d' bosons, is explained for $4 + 2 \rightarrow 10 + 2$ with the supergroup $SU(2,2|4)$ as in Sec. IV. This is slightly different in the other cases $(3 + 2 \rightarrow 11 + 2)$, $(5 + 2 \rightarrow 8 + 2)$, $(6 + 2 \rightarrow 11 + 2)$. The essential difference is the kappa supersymmetry described in Eq. (4.3), which seems to be present only in the case $4 + 2 \rightarrow 10 + 2$ but not in the others. The failure is due to the fact that once the upper and lower blocks of $\hat{L}$ are normalized as $i\frac{2}{s}\Gamma_{MN}L^{MN}$ and $i\frac{2}{s'}\Gamma_{IJ}L^{IJ}$ to satisfy the local bosonic symmetry $SO(d,2) \times SO(d')$, the kappa transformation δ_κ yields the structure $\frac{1}{s}L^{MN}(\Gamma_{MN}\xi) + \frac{1}{s'}L^{IJ}(\xi\Gamma_{IJ})$ instead of the one that appears in Eq. (4.4). When $s \neq s'$ this structure does not reduce to the $SO(d + d', 2)$ covariant dot products $\hat{X} \cdot \hat{X}, \hat{P}^- \cdot \hat{P}^-, \hat{P}^- \cdot \hat{X}$ and therefore cannot be cancelled by the variation of the $Sp(2,R)$ gauge fields. Thus, only the case of $4 + 2 \rightarrow 10 + 2$ seems to have the kappa supersymmetry given in Eq. (4.3). Because of the kappa supersymmetry in the case of $4 + 2 \rightarrow 10 + 2$, only 16 out of the 32 fermions in g are physical degrees of freedom, and their Clifford algebra is realized on 2^7 bosons and 2^7 fermions, which coincides with the physical spectrum of type-IIB supergravity in ten dimensions. By contrast, in the absence of kappa supersymmetry all of the 32 fermionic degrees of freedom in the group element g are physical and their quantized zero modes give a Clifford algebra realized on quantum states consisting of 2^{15} bosons and 2^{15} fermions, and these may be related to the first massive level of type-IIA supergravity or the supermembrane in 11 dimensions [9], or to the corresponding $AdS_4 \times S^7$, $AdS_7 \times S^4$ compactifications.

The reader may wonder whether other dimensions and/or supergroups may be used in a similar fashion. This is discussed in [14,16]. An essential point to consider is the $SO(d,2)$ applied on (X^M, P^M), taken in the spinor representation, versus the bosonic subgroup and the fermions in the supergroup G. The supergroups G of interest must contain $SO(d,2)$ in the spinor representation, namely, $spin(d,2)$, as a subgroup and its fermions must be in the spinor representation of $spin(d,2)$. In the cases discussed in this paper the spinor representation of $SO(d,2)$ for $d = 3, 4, 5, 6$, matched precisely with one of the blocks of the bosonic subgroup in G, while the other subgroup was the R-symmetry for N supersymmetries. When this is the case the gauging of $SO(d,2) \times$ (R–symmetry) can remove all the bosons from G and leave only the fermions in the correct spinor representation. This assures that the remaining degrees of freedom in the particle gauge describe a superparticle (or superstring) with usual properties. By contrast, when the bosonic subgroups in G contain more bosons than the ones in $spin(d,2)$, automatically there are more physical bosonic and fermionic degrees of freedom than just those of the superparticle or superstring. With the type of coupling $Str(\partial g g^{-1} L)$, with L in the spinor representation of $SO(d,2)$, one finds that the extra degrees of freedom in $g(\tau,\sigma)$ could be related to D-branes. Many models with brane degrees of freedom can be constructed in this enlarged scheme. One of the most interesting cases, aiming for a particle limit of 11-dimensional M-theory including branes, is constructed by using the supergroup $OSp(1|64)$ with $11 + 2$ dimensions [spinor representation of SO(11,2) is 64]. The particle case was briefly discussed in [14,16,27] and this is now generalized to string theory as the other cases in this paper.

As in the case of the particle, the 2T superstring in $d + 2$ dimensions discussed in this paper for several values of d can be gauge fixed into a variety of string-like 1T systems, with varying physical interpretation of the 1T dynamics. This phenomenon has so far not been investigated in string theory. It would be interesting to explore what one may learn about Yang-Mills theory or string theory from these dual holographic pictures, as well as from the unifying 2T theory that underlies them.

[16] $psu(2|2) + sl(2)$ has $3 + 3 + 3$ bosons in the adjoint representation of the $SU(2) \times SU(2) \times SL(2,R)$ subgroup, and eight real fermions in the complex $(2,2,1)$ representation. $V_{(3|8),2}$ is a doublet of the $SL(2,R)$ factor and under $SU(2) \times SU(2) \subset PSU(2|2)$ the symbol $(3|8)$ represents three real bosons in the representation $(3,0)$ and eight real fermions in the complex representation $(2,2)$. Therefore $V_{(3|8),2}$ contains $3 \times 2 = 6$ bosons and $8 \times 2 = 16$ fermions.

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